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Predictions for single spin asymmetries in $\ell p^\uparrow \to \pi X$ and $\gamma^* p^\uparrow \to \pi X$

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Abstract. Predictions for the single transverse spin asymmetry A_N in semi-inclusive DIS processes are given; non-negligible values of A_N may arise from spin effects in the fragmentation of a polarized quark into a final hadron with a transverse momentum $\boldsymbol{k}_\perp$ with respect to the jet axis, the so-called Collins effect. The elementary single spin asymmetry of the fragmenting quark has been fixed in a previous paper, by fitting data on $p^\uparrow p \to \pi X$, and by assuming that the QCD factorization theorem holds also when transverse momenta are taken into account. The predictions given here are based on the assumption that the Collins effect is the only cause of the observed single spin asymmetries in $p^\uparrow p \to \pi X$. Eventual spin and $\boldsymbol{k}_\perp$ dependences in the quark distribution functions, the so-called Sivers effect, are also discussed.

1 Introduction

A phenomenological description of single spin asymmetries in the inclusive production of hadrons in high energy processes, within the realm of pQCD, seems now indeed possible after the extensive amount of work which has recently been done [1–14]. Novel, hopefully universal, distribution and fragmentation functions [1–3,8,12] may give origin to sizable single spin hadronic observables; such functions can be measured in some processes and then used to give theoretical predictions in other cases.

This phenomenological program, based on data on $p^\uparrow p \to \pi X$ and $\bar{p}^\uparrow p \to \pi X$ [15], was initiated in [3,4] and continued in [5]; a similar attempt, together with a comprehensive theoretical discussion, is considered in [8].

Several possible origins of single spin asymmetries can be at work in $p^\uparrow p$ or $\bar{p}^\uparrow p$ processes: spin and intrinsic $\boldsymbol{k}_\perp$ dependences in the distribution functions of unpolarized quarks inside transversely polarized nucleons [1,3]; spin and intrinsic $\boldsymbol{k}_\perp$ dependences in the fragmentation functions of transversely polarized quarks into pions or other hadrons [2,5]; and very recently also possible spin and intrinsic $\boldsymbol{k}_\perp$ dependences in the distribution functions of transversely polarized quarks inside unpolarized nucleons have been suggested [12].

The first two possibilities above have been considered respectively in [4] and [5] and explicit expressions of the relevant elementary functions have been obtained:

$$\Delta^N f_{a/p}(x_a, \boldsymbol{k}_{\perp a}) = \hat{f}_{a/p^\uparrow}(x_a, \boldsymbol{k}_{\perp a}) - \hat{f}_{a/p^\downarrow}(x_a, \boldsymbol{k}_{\perp a}), \quad (1)$$

i.e. the difference between the density numbers of partons a, with all possible polarizations, longitudinal momentum fractions x_a and intrinsic transverse momenta $\boldsymbol{k}_{\perp a}$, inside a transversely polarized proton, and

$$\Delta^N D_{h/a}(z, \boldsymbol{k}_{\perp h}) = \hat{D}_{h/a^\uparrow}(z, \boldsymbol{k}_{\perp h}) - \hat{D}_{h/a^\downarrow}(z, \boldsymbol{k}_{\perp h}), \quad (2)$$

i.e. the difference between the density numbers of hadrons h, with longitudinal momentum fraction z and transverse momentum $\boldsymbol{k}_{\perp h}$ inside a jet having its origin in the fragmentation of a transversely polarized parton a.

These two functions are strictly related to the functions denoted by $f_{1T}^\perp$ and $H_1^\perp$ in [9], as is further discussed in [14].

We consider here the DIS processes $\ell p^\uparrow \to \pi X$ and $\gamma^* p^\uparrow \to \pi X$, with unpolarized leptons; however, the same results hold in case of longitudinally polarized leptons; it is only crucial for the proton spin to be orthogonal to the scattering plane. As was remarked in [5] and [16] we do not expect any spin effect in the nucleon distribution functions in such DIS processes, because these should be suppressed by the necessary initial state interactions which mean higher powers of $\alpha_{\rm em}$. There remains then the only possibility of the quark fragmentation single spin asymmetry, $\Delta^N D_{h/a}(z, \boldsymbol{k}_\perp) \neq 0$, the so-called Collins effect [2].

Such an effect was studied in [5] where, assuming that it is the only cause of the observed single spin asymmetry in $p^\uparrow p \to \pi X$, an explicit expression of $\Delta^N D_{h/a}(z, \boldsymbol{k}_\perp)$, based on a simple parametrization, was obtained. By using this expression here we are able to give predictions for the single spin asymmetry

$$A_N = \frac{\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow}{\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow} = \frac{\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow}{2\mathrm{d}\sigma^{\mathrm{unp}}} \tag{3}$$

in DIS processes.

In the next section we will consider the process $\ell p^\uparrow \to \pi X$, in which the final lepton need not be detected, in complete analogy with the $p^\uparrow p \to \pi X$ process; parity invariance allows A_N to be different from zero only if the nucleon spin has a component perpendicular to the scattering plane. This is not the case for longitudinally polarized protons. However, also for such spin configurations, one could have a single spin asymmetry by considering the semi-inclusive process $\ell p^\uparrow \to \ell\pi X$ or $\gamma^* p^\uparrow \to \pi X$; also a longitudinally polarized proton can have a non-zero spin component perpendicular to the scattering plane, which is now the γ^*–π plane. Predictions for such a process will be given in Sect. 3.

In Sect. 4 we consider the hypothetical possibility that the Sivers effect, i.e. the function $\Delta^N f$ of (1), might be at work also in DIS processes [19], despite the argument given above about initial state interactions, and we compute A_N in $\ell p^\uparrow \to \pi X$ due to such an effect using the expressions of $\Delta^N f$ derived in [4]; we do this in order to compare the size and x_F dependence of A_N eventually originating from the Sivers effect with those of A_N originating from the Collins effect. Comments and conclusions are presented in Sect. 5.

2 Single spin asymmetry A_N for $\ell p^\uparrow \to \pi X$

The single transverse spin asymmetry (3) for the high energy process $\ell p^\uparrow \to \pi X$ in which a final pion is detected with a large p_T is given, assuming a straightforward generalization of the QCD factorization theorem [2] to the case in which the parton transverse motion is taken into account and considering $\boldsymbol{k}_\perp$ dependences only in the fragmentation process, by [5,16]

$$\begin{aligned} 2A_N \frac{E_\pi \mathrm{d}^3\sigma^{\mathrm{unp}}}{\mathrm{d}^3\boldsymbol{p}_\pi} &= \frac{E_\pi \mathrm{d}^3\sigma^{\ell+p^\uparrow\to\pi+X}}{\mathrm{d}^3\boldsymbol{p}_\pi} \\ &\quad - \frac{E_\pi \mathrm{d}^3\sigma^{\ell+p^\downarrow\to\pi+X}}{\mathrm{d}^3\boldsymbol{p}_\pi} \\ &= \sum_q \int \frac{\mathrm{d}x}{\pi z} \mathrm{d}^2\boldsymbol{k}_\perp \Delta_{\mathrm{T}} q(x) \Delta_{NN}\hat\sigma^q(x,\boldsymbol{k}_\perp) \\ &\quad \times \left[\hat D_{\pi/q^\uparrow}(z,\boldsymbol{k}_\perp) - \hat D_{\pi/q^\uparrow}(z,-\boldsymbol{k}_\perp)\right], \end{aligned} \tag{4}$$

where $\Delta_{\mathrm{T}} q$ is the polarized number density for transversely spinning quarks q, also denoted by h_1 [17], and $\Delta_{NN}\hat\sigma^q$ is the elementary cross-section double spin asymmetry

$$\Delta_{NN}\hat\sigma^q = \frac{\mathrm{d}\hat\sigma^{\ell q^\uparrow\to\ell q^\uparrow}}{\mathrm{d}\hat t} - \frac{\mathrm{d}\hat\sigma^{\ell q^\uparrow\to\ell q^\downarrow}}{\mathrm{d}\hat t}. \tag{5}$$

Here $\uparrow$ and $\downarrow$ refer to directions "up" and "down" with respect to the scattering plane: $\uparrow$ is parallel to $\boldsymbol{p}_\ell \times \boldsymbol{p}_\pi$. Notice that (4) holds both for unpolarized and longitudinally polarized leptons.

Following the same procedure as in [5] we write (4) as

$$\begin{aligned} &\frac{E_\pi \mathrm{d}^3\sigma^{\ell+p^\uparrow\to\pi+X}}{\mathrm{d}^3\boldsymbol{p}_\pi} - \frac{E_\pi \mathrm{d}^3\sigma^{\ell+p^\downarrow\to\pi+X}}{\mathrm{d}^3\boldsymbol{p}_\pi} \\ &= \sum_q \int \frac{\mathrm{d}x}{\pi z} \mathrm{d}^2\boldsymbol{k}_\perp P^{q/p^\uparrow} f_{q/p}(x) \\ &\times [\Delta_{NN}\hat\sigma^q(x,\boldsymbol{k}_\perp) - \Delta_{NN}\hat\sigma^q(x,-\boldsymbol{k}_\perp)] \\ &\times \Delta^N D_{\pi/q}(z,\boldsymbol{k}_\perp), \end{aligned} \tag{6}$$

where we have used (2) and where the integration on $\boldsymbol{k}_\perp$ now runs only over one half-plane of its components. We have also used $\Delta_{\mathrm{T}} q(x) = h_1(x) = P^{q/p^\uparrow} f_{q/p}(x)$ where $P^{q/p^\uparrow}$ is the polarization of the quark q inside the transversely polarized proton $p^\uparrow$; it is twice the average value of the $\uparrow$ component of the quark spin.

We now proceed as in [5] with the assumption that the dominant contribution is given by valence quarks, which is certainly correct for large x_F pions which originate from large x quarks; for this reason we also assume values of $P^{q/p^\uparrow}$ which do not depend on x. We take into account only the leading $\boldsymbol{k}_\perp$ contributions which originate from $\boldsymbol{k}_\perp = \boldsymbol{k}_\perp^0$ where $\boldsymbol{k}_\perp^0$ lies in the overall scattering plane and its magnitude equals the average value of $\langle \boldsymbol{k}_\perp^2\rangle^{1/2} = k_\perp^0(z)$. That is, we have

$$\begin{aligned} &\frac{E_\pi \mathrm{d}^3\sigma^{\ell+p^\uparrow\to\pi+X}}{\mathrm{d}^3\boldsymbol{p}_\pi} - \frac{E_\pi \mathrm{d}^3\sigma^{\ell+p^\downarrow\to\pi+X}}{\mathrm{d}^3\boldsymbol{p}_\pi} \\ &\simeq \sum_q \int \frac{\mathrm{d}x}{\pi z} P^{q/p^\uparrow} f_{q/p}(x) \\ &\times [\Delta_{NN}\hat\sigma^q(x,+\boldsymbol{k}_\perp^0) - \Delta_{NN}\hat\sigma^q(x,-\boldsymbol{k}_\perp^0)] \Delta^N D_{\pi/q}(z,k_\perp^0). \end{aligned} \tag{7}$$

The value of z is fixed as function of x and $\boldsymbol{k}_\perp$ by energy-momentum conservation in the elementary scattering.

The average value $k_\perp^0$ of the transverse momentum of charged pions inside jets does in general depend on z; the z dependence was measured at LEP [18] and we use a fit to their data points [5]:

$$\frac{k_\perp^0(z)}{M} = 0.61 z^{0.27}(1-z)^{0.20}, \tag{8}$$

with $M = 1\,\mathrm{GeV}/c^2$.

The explicit expression of $\Delta^N D_{\pi/q}$ obtained in [5] by fitting the data on $p^\uparrow p \to \pi X$ [15] is, for valence quarks,

$$\begin{aligned} &\Delta^N D_{\pi^+/u}(z,k_\perp^0) = \Delta^N D_{\pi^-/d}(z,k_\perp^0) = \Delta^N D_{\mathrm{val}}(z,k_\perp^0) \\ &= 2\Delta^N D_{\pi^0/u}(z,k_\perp^0) = 2\Delta^N D_{\pi^0/d}(z,k_\perp^0) \\ &\qquad = -\frac{k_\perp^0(z)}{M} 0.22 z^{2.33}(1-z)^{0.24} \end{aligned} \tag{9}$$

for $z \le 0.97742$ and

$$\Delta^N D_{\mathrm{val}}(z,k_\perp^0) = -2D_{\mathrm{val}}(z) = -2\times 1.102 z^{-1}(1-z)^{1.2} \tag{10}$$

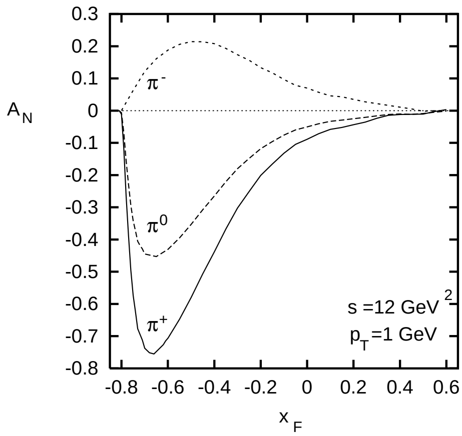

Fig. 1. Predictions for the single spin asymmetry A_N in the DIS process $lp^\uparrow \to \pi X$ as a function of x_F: the solid line refers to π^+, the dashed line to π^0 and the dotted line to π^-. We assume that only the Collins effect is active and the function $\Delta^N D(z, k_\perp)$ needed to calculate A_N is derived from a fit to the E704 experimental data on $p^\uparrow p \to \pi X$ [5]. The value of s is chosen as a planned CEBAF value, $s = 12\,\mathrm{GeV}^2$, and the transverse momentum of the pion is fixed to $p_\mathrm{T} = 1.0\,\mathrm{GeV}$

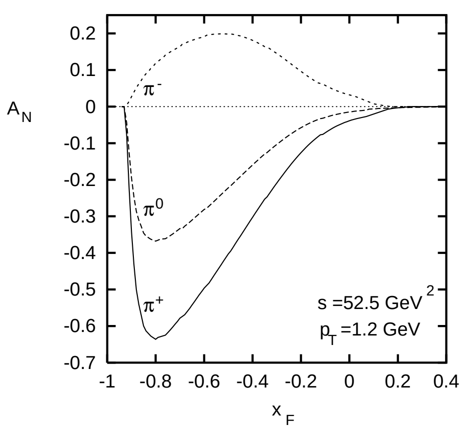

Fig. 2. The same as in Fig. 1 for typical kinematical values of the HERMES experiment, $s = 52.5\,\mathrm{GeV}^2$, and $p_\mathrm{T} = 1.2\,\mathrm{GeV}$

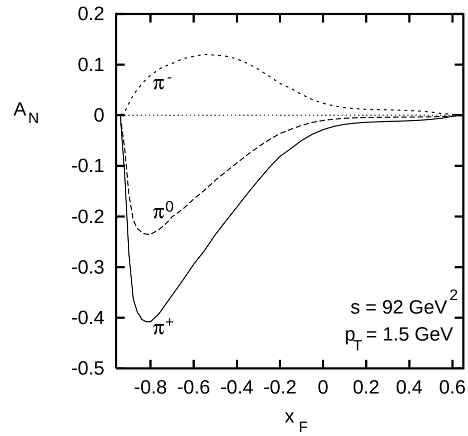

Fig. 3. The same as in Fig. 1 at the SLAC energy $s = 92\,\mathrm{GeV}^2$, with the transverse momentum of the pion fixed to $p_\mathrm{T} = 1.5\,\mathrm{GeV}$

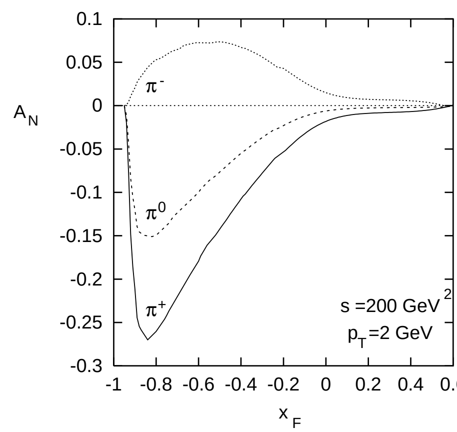

Fig. 4. The same as in Fig. 1 for the SMC energy value $s = 200\,\mathrm{GeV}^2$, and a transverse momentum of the pion of $p_\mathrm{T} = 2\,\mathrm{GeV}$. Notice that $s = 400\,\mathrm{GeV}^2$ would yield a very similar result

for $z > 0.97742$. The values of $P^{u/p^\uparrow}$ and $P^{d/p^\uparrow}$, always from the fit of [5], are

$$P^{u/p^\uparrow} = \frac{2}{3}, \qquad P^{d/p^\uparrow} = -0.88; \tag{11}$$

notice that only their ratio $P_{u/d} = P^{u/p^\uparrow}/P^{d/p^\uparrow} = -0.76$ has a physical meaning in that the overall normalization of the quark polarizations can always be absorbed by the overall normalization of $\Delta^N D_{\pi/q}$.

As we commented in [5] both the expression of $\Delta^N D_\mathrm{val}$ and the value of $P_{u/d}$ resulting from fitting the $p^\uparrow p \to \pi X$ data – assuming spin and $\boldsymbol{k}_\perp$ effects only in the fragmentation process – are somewhat surprising and a bit extreme: $\Delta^N D_\mathrm{val}$ has to saturate at large z the necessary inequality $|\Delta^N D_\mathrm{val}(z, k_\perp^0)| \le 2D_\mathrm{val}(z)$ and $P_{u/d}$ shows an opposite (as expected) but larger in magnitude (unexpected) polarization of the d valence quark with respect to the u valence quark inside a polarized proton. It might be that relevant or even dominant contributions to A_N in $p^\uparrow p \to \pi X$ processes originate from quark distribution functions, (1), the so-called Sivers effect [1,3,4].

Nevertheless, we assume here that the Collins effect alone is responsible for the observed single spin asymmetries and use (8)–(11) into (7) to obtain predictions for A_N, (3). The unpolarized cross section

$$\frac{E_\pi \mathrm{d}^3\sigma^\mathrm{unp}}{\mathrm{d}^3\boldsymbol{p}_\pi} = \sum_q \int \frac{\mathrm{d}x}{\pi z} f_{q/p}(x) \frac{\mathrm{d}\hat{\sigma}^{\ell q \to \ell q}}{\mathrm{d}\hat{t}}(x) D_{\pi/q}(z) \tag{12}$$

is computed using the same distribution and fragmentation functions as in [5].

Our results are shown in Figs. 1–4, where we plot A_N as a function of x_F at different center of mass energies typical of experiments running or planned at Jefferson Lab (CEBAF), DESY (HERMES), SLAC (E155) and CERN (SMC, COMPASS), respectively; p_T is fixed and ranges, according to the different cases, from 1 to $2\,\mathrm{GeV}/c$. The asymmetry turns out to be large and sizable at large values of x_F in all cases: we can unambiguously conclude that, if

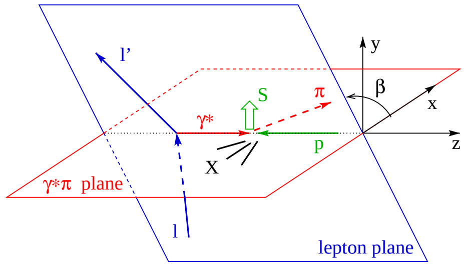

Fig. 5. Kinematical and spin configurations adopted for the computation of A_N in $\gamma^* p^\uparrow \to \pi X$ processes, (20) of the text

the Collins effect is the main cause of the single transverse spin asymmetries observed in $p^\uparrow p \to \pi X$ [15], a similarly large asymmetry should be observed in lepto-production processes, $\ell p^\uparrow \to \pi X$.

Data on A_N in such processes are not available yet, although they might soon be obtainable from SMC and SLAC where DIS processes with transversely polarized protons are studied. Protons are longitudinally polarized at HERMES and Jlab experiments and parity invariance does not allow a single proton spin asymmetry in $\ell p^\rightarrow \to \pi X$; however, as we said in the Introduction and will discuss in the next section, even in such a case one could have a single transverse spin asymmetry considering the $\gamma^* p^\uparrow \to \pi X$ reaction.

3 Single spin asymmetry A_N for $\gamma^* p^\uparrow \to \pi X$

Let us consider now the double inclusive process $\ell p^\uparrow \to \ell \pi X$ in which also the final lepton is detected, which allows one to fix the energy and momentum of the virtual photon exchanged between the lepton and the nucleon; one can then consider the center of mass process $\gamma^* p^\uparrow \to \pi X$. A longitudinally polarized nucleon, i.e. a nucleon with its spin along the lepton direction in the laboratory frame, may have a transverse spin component with respect to the γ^*–π plane.

We fix our kinematics in Fig. 5; we have defined the $\gamma^* p^\uparrow \to \pi X$ scattering plane as the xz plane; the spin $\uparrow$ ($\downarrow$) is then along the $+y$ ($-y$) direction. The angle between the ℓ–ℓ' and the γ^*–π planes is β. We shall give values of the single spin asymmetry A_N, (3), for this configuration.

The analogue of (4) is now

$$
\begin{aligned}
2A_N \frac{\mathrm{d}\sigma^{\gamma^* p \to \pi X}}{\mathrm{d}x \mathrm{d}Q^2 \mathrm{d}z \mathrm{d}^2 p_\mathrm{T}} &= \frac{\mathrm{d}\sigma^{\gamma^* p^\uparrow \to \pi X}}{\mathrm{d}x \mathrm{d}Q^2 \mathrm{d}z \mathrm{d}^2 p_\mathrm{T}} - \frac{\mathrm{d}\sigma^{\gamma^* p^\downarrow \to \pi X}}{\mathrm{d}x \mathrm{d}Q^2 \mathrm{d}z \mathrm{d}^2 p_\mathrm{T}} \\
&= \sum_q P^{q/p^\uparrow} f_{q/p}(x) \left[\frac{\mathrm{d}\hat\sigma^{\gamma^* q^\uparrow \to q^\uparrow}}{\mathrm{d}Q^2} - \frac{\mathrm{d}\hat\sigma^{\gamma^* q^\uparrow \to q^\downarrow}}{\mathrm{d}Q^2} \right] \\
&\times \left[\hat D_{\pi/q^\uparrow}(z, \boldsymbol{p}_\mathrm{T}) - \hat D_{\pi/q^\uparrow}(z, -\boldsymbol{p}_\mathrm{T}) \right], \qquad (13)
\end{aligned}
$$

where x and Q^2 are the usual DIS variables and where the unpolarized cross section is

$$
\frac{\mathrm{d}\sigma^{\gamma^* p \to \pi X}}{\mathrm{d}x \mathrm{d}Q^2 \mathrm{d}z \mathrm{d}^2 p_\mathrm{T}} = \sum_q f_{q/p}(x) \frac{\mathrm{d}\hat\sigma^{\gamma^* q \to q}}{\mathrm{d}Q^2} \hat D_{\pi/q}(z, \boldsymbol{p}_\mathrm{T}). \qquad (14)
$$

Notice that in the γ^*–p c.m. frame, neglecting the transverse motion of the quarks inside the polarized proton – that is, taking into account only the Collins effect – the transverse momentum $\boldsymbol{p}_\mathrm{T}$ of the produced pion is the same as the intrinsic $\boldsymbol{k}_\perp$ of the pion relative to fragmenting quark direction. The angle between the quark transverse spin direction and $\boldsymbol{k}_\perp$ – sometimes referred to as the Collins angle – is fixed and equals $\pi/2$ in the configuration of Fig. 5.

The cross section for the $\gamma^* q \to q$ process, with unpolarized quarks, is given by

$$
\begin{aligned}
&\frac{\mathrm{d}\hat\sigma^{\gamma^* q \to q}}{\mathrm{d}Q^2} \qquad (15) \\
&= C \frac{1}{2} \sum_{\lambda_q, \lambda_{q'}} \sum_{\lambda_\gamma, \lambda'_\gamma} M_{\lambda_{q'}; \lambda_\gamma, \lambda_q} M^*_{\lambda_{q'}; \lambda'_\gamma, \lambda_q} \rho_{\lambda_\gamma, \lambda'_\gamma}(\gamma^*) \\
&\equiv C \frac{1}{2} \sum_{\lambda_q, \lambda_{q'}} \sum_{\lambda_\gamma, \lambda'_\gamma} \langle \lambda_{q'} | \lambda_\gamma, \lambda_q \rangle \langle \lambda_{q'} | \lambda'_\gamma, \lambda_q \rangle^* \rho_{\lambda_\gamma, \lambda'_\gamma}(\gamma^*),
\end{aligned}
$$

where the M's are the helicity amplitudes for the $\gamma^* q \to q$ process, C contains kinematical and flux factors which cancel out in the ratio giving A_N and $\rho(\gamma^*)$ is the helicity density matrix of the virtual photon emitted by the lepton; its explicit expression depends on the DIS variables and can be found, for example, in [20]. Analogously one has, for transversely polarized quarks,

$$
\begin{aligned}
&\frac{\mathrm{d}\hat\sigma^{\gamma^* q^\uparrow \to q^{\uparrow(\downarrow)}}}{\mathrm{d}Q^2} \\
&= C \sum_{\lambda_\gamma, \lambda'_\gamma} \langle \uparrow (\downarrow) | \lambda_\gamma, \uparrow \rangle \langle \uparrow (\downarrow) | \lambda'_\gamma, \uparrow \rangle^* \rho_{\lambda_\gamma, \lambda'_\gamma}(\gamma^*). \qquad (16)
\end{aligned}
$$

In terms of helicity states the transverse spins "up" and "down" are given, for the initial (q) and final (q') quarks, by

$$
\begin{aligned}
|q; \uparrow\rangle &= \frac{1}{\sqrt{2}} \left(|+\rangle - \mathrm{i} |-\rangle \right), \\
|q'; \uparrow, \downarrow\rangle &= \frac{1}{\sqrt{2}} \left(\pm |+\rangle + \mathrm{i} |-\rangle \right). \qquad (17)
\end{aligned}
$$

One finds

$$
\frac{\mathrm{d}\hat\sigma^{\gamma^* q \to q}}{\mathrm{d}Q^2} = C e^2 e_q^2 Q^2 \frac{1 + (1-y)^2}{(2-y)^2} \qquad (18)
$$

and

$$
\frac{\mathrm{d}\hat\sigma^{\gamma^* q^\uparrow \to q^\uparrow}}{\mathrm{d}Q^2} - \frac{\mathrm{d}\hat\sigma^{\gamma^* q^\uparrow \to q^\downarrow}}{\mathrm{d}Q^2} = 2 C e^2 e_q^2 Q^2 \frac{1-y}{(2-y)^2} \cos 2\beta, \qquad (19)
$$

where $y = Q^2/xs$; notice that a dependence on the angle β (see Fig. 5) remains in the elementary polarized process.

Insertion of (18) and (19) into (13) and (14) yields

$$
\begin{aligned}
A_N^{\gamma^* p^\uparrow \to \pi X}(x, Q^2, z, p_\mathrm{T}) &= \frac{(1-y)\cos 2\beta}{[1 + (1-y)^2]} \\
&\times \frac{\sum_q e_q^2 P^{q/p^\uparrow} f_{q/p}(x) \Delta^N D_{\pi/q}(z, p_\mathrm{T})}{\sum_q e_q^2 f_{q/p}(x) D_{\pi/q}(z)}. \qquad (20)
\end{aligned}
$$

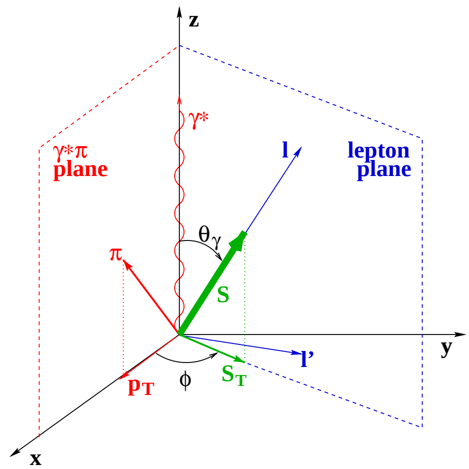

Fig. 6. The planes γ^*–π and ℓ–ℓ' in the reference frame where the γ^* moves along the z-axis and the nucleon at rest is longitudinally polarized, i.e. its spin is parallel to the ℓ-direction of motion. The nucleon spin component perpendicular to the γ^*–π plane is $S\sin\theta_\gamma\sin\Phi = S_{\rm T}\sin\Phi$ and the Collins angle between $\boldsymbol{S}_{\rm T}$ and $\boldsymbol{p}_{\rm T}$ is Φ

We give numerical estimates of $A_N/|\cos 2\beta|$ using $\Delta^N D_{\pi/q}$ and $P^{q/p^\uparrow}$ from (9)–(11). The average $p_{\rm T}$ value is given, at any z, by (8); the $p_{\rm T}$ dependence in the fragmentation function in the denominator of (20) is neglected.

The actual asymmetry A_N measurable in processes initiated by longitudinally polarized protons – i.e. protons with spin parallel to the lepton direction – is smaller than that given in (20), which assumes a $P = 100\%$ nucleon polarization perpendicular to the γ^*–π scattering plane. The real kinematical situation is shown in Fig. 6, from which one sees that the observed asymmetry is given by (20) multiplied by $P\sin\theta_\gamma\sin\Phi$ (the component of the actual target polarization P perpendicular to the production plane) and in which one drops the factor $\cos 2\beta$. Some preliminary data from HERMES and SMC show the Φ dependence of A_N [21,22].

In Figs. 7 and 8 we plot the value of $A_N(x, Q^2, z)$ for CEBAF and HERMES at planned or running energies, as a function of z, for several values of x and at fixed values of Q^2. These plots show that the asymmetry is large, for large x values, but only at very large z values, which might be difficult to reach experimentally.

The z dependence of A_N is dominated by the z dependence of $\Delta^N D_{\pi/q}/D_{\pi/q}$; actually, in all cases in which only one flavour q contributes dominantly, (20) simplifies to

$$A_N^{\gamma^* p^\uparrow\to\pi X}(y, z, p_{\rm T}) \simeq \frac{(1-y)\cos 2\beta}{[1+(1-y)^2]}\frac{P^{q/p^\uparrow}\Delta^N D_{\pi/q}(z, p_{\rm T})}{D_{\pi/q}(z)}. \quad (21)$$

It is then clear why A_N is small at small z values and is large and constant for $z \geq 0.977$: this is due to the behavior of $\Delta^N D_{\rm val}$ resulting from fitting the $p^\uparrow p \to \pi X$ data; see Fig. 3 of [5]. In particular the saturated behavior of $\Delta^N D_{\rm val}$ at large z, (10), forces the large value of A_N when

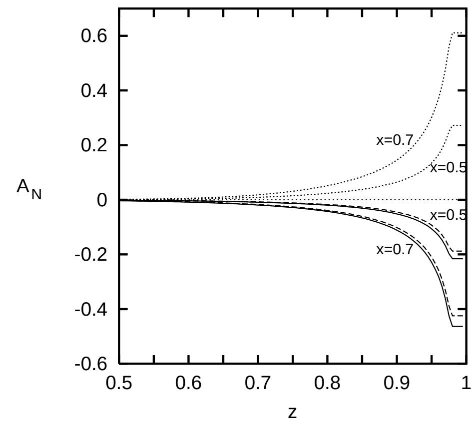

Fig. 7. Prediction for the single spin asymmetry $A_N/|\cos 2\beta|$, (20) of the text, for pion production in the process $\gamma^* p^\uparrow \to \pi X$ as a function of z. Here we have taken $Q^2 = 5.0\,\mathrm{GeV}^2$ and $s = 12\,\mathrm{GeV}^2$. The two different sets of curves correspond to either $x = 0.5$ or $x = 0.7$. The solid and dashed lines in the negative plane are for π^+ and π^0, respectively. The dotted curves in the positive plane correspond to π^-. For $z \geq 0.97742$ the single spin asymmetry is constant because of the constraint $\Delta^N D = -2D$ [5]. The z-range has been restricted to 0.5–1.0 for aesthetic reasons only (A_N is zero between $z = 0.0$ and $z = 0.5$). Notice how the single spin asymmetry for π^0 is almost the same as that for π^+

$z \to 1$. All these are typical features of the Collins effect, assuming that it is the only origin of single transverse spin asymmetries in $p^\uparrow p \to \pi X$ processes, and it would be very interesting to check them experimentally.

4 Single spin asymmetry A_N for $\ell p^\uparrow \to \pi X$ with the Sivers effect

The Sivers effect is forbidden by time reversal invariance for free particles [2]; however, one can avoid this by invoking some soft initial state interactions between the incoming particles [3,12]. This is plausible for pp interactions or, in general, hadron–hadron interactions where many soft gluons might be exchanged, or some external gluonic fields should be present; the assumption of the validity of the factorization theorem also in this case allows then to compute A_N due to Sivers effect [3,4].

We cannot use such an argument in ℓN processes because initial state interactions would now lead to strong suppressions of $\mathcal{O}(\alpha_{\rm em})$. It has been suggested [19] that some spin–isospin interactions might avoid the time reversal problem in chiral models, with quarks moving in a background of chiral fields, with a spin–isospin interaction which mixes states of different flavors under time reversal.

We do not claim here the definite validity of the above suggestion, which would require further developments and for which we have no explicit model; we simply assume that the Sivers effect can be at work also in ℓp interactions and we consistently compute A_N in $\ell p^\uparrow \to \pi X$ processes due solely to this origin. Our aim is that of comparing the values and features of A_N due to the Sivers effect to

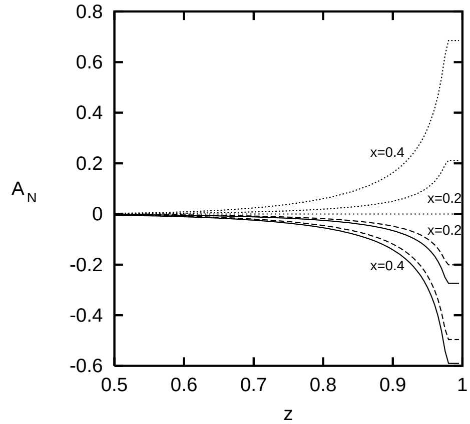

Fig. 8. The same as in Fig. 7 with typical HERMES kinematical values: $Q^2 = 8.0\,\mathrm{GeV}^2$ and $s = 52.4\,\mathrm{GeV}^2$. The two different sets of curves correspond to either $x = 0.2$ or $x = 0.4$ (notice that the cut in x for HERMES is $0.02 \le x \le 0.4$)

the values and features of A_N due to the Collins effect, as computed in Sect. 1, in order to eventually distinguish between the two cases, independently of their theoretical plausibilities.

If we assume that only the Sivers effect is at work, (4) changes into

$$\begin{aligned} 2A_N \frac{E_\pi \mathrm{d}^3\sigma^{\mathrm{unp}}}{\mathrm{d}^3\boldsymbol{p}_\pi} &= \frac{E_\pi \mathrm{d}^3\sigma^{\ell+p^\uparrow\to\pi+X}}{\mathrm{d}^3\boldsymbol{p}_\pi} \\ &\quad - \frac{E_\pi \mathrm{d}^3\sigma^{\ell+p^\downarrow\to\pi+X}}{\mathrm{d}^3\boldsymbol{p}_\pi} \\ &= \sum_q \int \frac{\mathrm{d}x}{\pi z} \mathrm{d}^2\boldsymbol{k}_\perp \Delta^N f_{q/p}(x, \boldsymbol{k}_\perp) \\ &\quad \times \frac{\mathrm{d}\hat{\sigma}^{\ell q\to\ell q}}{\mathrm{d}\hat{t}}(x, \boldsymbol{k}_\perp) D_{\pi/q}(z) \end{aligned} \tag{22}$$

where $\mathrm{d}\sigma^{\mathrm{unp}}$ is the same as in (12). Again, (22) holds both for unpolarized and longitudinally polarized leptons: the proton spin has to be orthogonal to the scattering plane.

We proceed in the same way as in Sect. 1, and [3,4], to obtain, in analogy to (7)

$$\begin{aligned} &\frac{E_\pi \mathrm{d}^3\sigma^{\ell+p^\uparrow\to\pi+X}}{\mathrm{d}^3\boldsymbol{p}_\pi} - \frac{E_\pi \mathrm{d}^3\sigma^{\ell+p^\downarrow\to\pi+X}}{\mathrm{d}^3\boldsymbol{p}_\pi} \\ &\simeq \sum_q \int \frac{\mathrm{d}x}{\pi z} \Delta^N f_{q/p}(x, k_\perp^0)[\mathrm{d}\hat{\sigma}^{\ell q\to\ell q}(x, +\boldsymbol{k}_\perp^0) \\ &- \mathrm{d}\hat{\sigma}^{\ell q\to\ell q}(x, -\boldsymbol{k}_\perp^0)] D_{\pi/q}(z). \end{aligned} \tag{23}$$

The function $\Delta^N f_{q/p}(x, k_\perp^0)$ can be found in [4], where it was derived for u and d valence quarks by fitting the data on $p^\uparrow p \to \pi X$ (notice that, as commented in [5], the values of N_a given in (9) of [4] should be multiplied by a factor 4):

$$\Delta^N f_{u/p}(x, k_\perp^0) = \frac{k_\perp^0(x)}{M} 14.72 x^{1.34}(1-x)^{3.58}, \tag{24}$$

$$\Delta^N f_{d/p}(x, k_\perp^0) = -\frac{k_\perp^0(x)}{M} 4.96 x^{0.76}(1-x)^{4.14}, \tag{25}$$

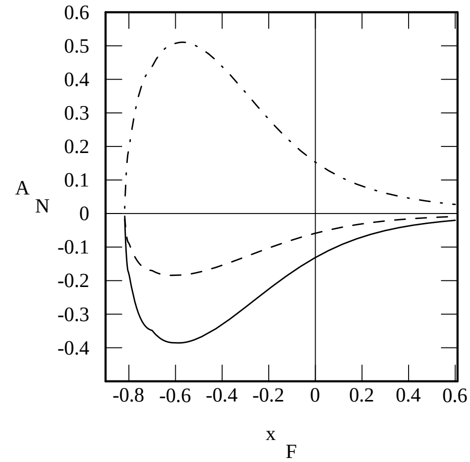

Fig. 9. Predictions for the single spin asymmetry A_N in the DIS process $lp^\uparrow \to \pi X$ as a function of x_F: the solid line refers to π^+, the dashed line to π^0 and the dotted-dashed line to π^-. We hypothetically assume that only the Sivers effect is active and the function $\Delta^N f(x, k_\perp)$ needed to calculate A_N is derived from a fit to E704 experimental data on $p^\uparrow p \to \pi X$ [4]. The value of s is chosen as a planned CEBAF value, $s = 12\,\mathrm{GeV}^2$, and the transverse momentum of the pion is fixed to $p_\mathrm{T} = 1.0\,\mathrm{GeV}$

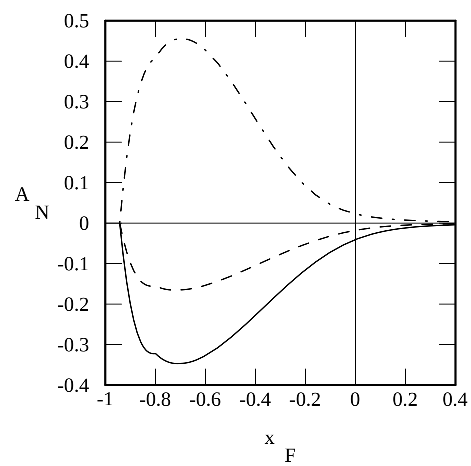

Fig. 10. The same as in Fig. 9 for typical kinematical values of the HERMES experiment, $s = 52.5\,\mathrm{GeV}^2$, and $p_\mathrm{T} = 1.2\,\mathrm{GeV}$

with

$$\frac{k_\perp^0(x)}{M} = 0.47 x^{0.68}(1-x)^{0.48}. \tag{26}$$

From (22)–(26) and (12) we can compute A_N as hypothetically given by the Sivers effect alone; the results, at the same energy and p_T values of the similar results obtained from Collins effects, Figs. 1–4, are shown in Figs. 9–12.

Also in this case A_N turns out to be large and detectable. The x_F dependences and the maximum values are clearly different from those generated by the Collins effect: A_N is smoother and less pronounced for π^+ and π^0 and steeper and larger for π^-, the opposite of what is shown in Figs. 1–4 where π^+ and π^0 have a stronger dependence on x_F and higher absolute values than π^-.

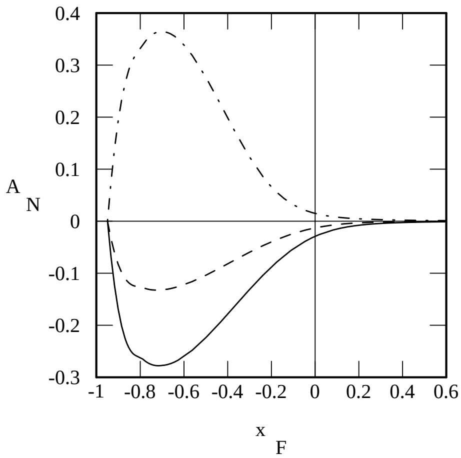

Fig. 11. The same as in Fig. 9 at SLAC energy, $s = 92\,\mathrm{GeV}^2$, with the transverse momentum of the pion fixed to $p_\mathrm{T} = 1.5\,\mathrm{GeV}$

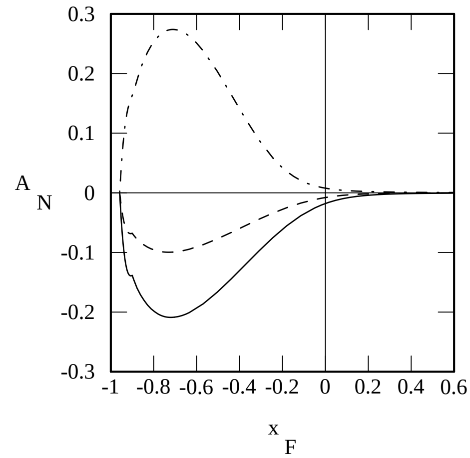

Fig. 12. The same as in Fig. 9 for the SMC energy value $s = 200\,\mathrm{GeV}^2$, and a transverse momentum of the pion of $p_\mathrm{T} = 2\,\mathrm{GeV}$

In particular, the Sivers effect would produce the biggest (in magnitude) asymmetries for π^-'s, whereas the Collins effect does so for π^+'s.

5 Comments and conclusions

Single transverse spin asymmetries have been observed in $p^\uparrow p \to \pi X$ and $\bar{p}^\uparrow p \to \pi X$ processes [15] in a kinematical region where the unpolarized cross section is well described by pQCD and the factorization theorem at leading twist: these measured large values of A_N are very surprising because the same pQCD and factorization theorem at leading twist predict negligible values of A_N. They have to be understood and checked with further experiments.

Several possible explanations have been suggested within QCD [1–14] or even classical models [23], and indeed phenomenological consistent descriptions of single spin asymmetries in several processes now seem possible and should be pursued in order to understand the basic underlying mechanisms which yield them. We have studied two possible mechanisms [3–5], the so-called Sivers and Collins effects, respectively described by the new $\boldsymbol{k}_\perp$ and spin dependent functions of (1) and (2). By fitting the available data we have deduced simple explicit expressions for $\Delta^N f_{q/p}(x, k_\perp^0)$ and $\Delta^N D_{\pi/q}(z, k_\perp^0)$.

We have considered here the inclusive production of pions in lepton–nucleon interactions in the process $\ell p^\uparrow \to \pi X$ in complete analogy to $p^\uparrow p \to \pi X$. Independently of the details of the proposed mechanisms to generate single spin asymmetries, the mere fact of measuring A_N in such a process is significant and important. As we said, for several reasons we expect that single spin asymmetry in ℓN interactions can only originate from single spin effects in the fragmentation of a transversely polarized quark into the final pion; if this is true also for $p^\uparrow p \to \pi X$, as some authors think, then A_N should be of similar nature in the $\ell p^\uparrow$ and $pp^\uparrow$ (or $p^\uparrow p$) interactions. The different elementary dynamics in the two cases – $\ell q \to \ell q$ versus QCD dynamics like $qq \to qq$ or $qg \to qg$ – should result in (very interesting) differences in the shape of A_N, but not in the size of its magnitude.

In Figs. 1–4 we have presented our predictions for A_N in $\ell p^\uparrow \to \pi X$ processes exactly under the above assumption: that only the Collins effect can be responsible for some single transverse spin dependence. If the data agree with our predictions it would confirm this assumption and the consistence of our phenomenological approach to the computations of inclusive single spin asymmetries; if not, it might be that other effects, like the Sivers effect, are responsible for A_N in hadron–hadron collisions. Data might already be available from SMC and SLAC experiments, where nucleons can be transversely polarized, and will be available in the near future from other experiments which have now only longitudinally polarized nucleons.

Single transverse spin asymmetries may be measurable also in the case of longitudinally polarized nucleons provided one looks at the double inclusive process, $\ell p^\uparrow \to \ell \pi X$ from which one can reconstruct the $\gamma^* p^\uparrow \to \pi X$ reaction, which, in general, occurs in a plane different from the ℓ–ℓ' plane where the longitudinal nucleon spin lies (Fig. 5). This further selection of events – isolating those such that the proton polarization vector has a significant component perpendicular to the γ^*–π scattering plane – might greatly reduce the experimental available information; also, our predictions, Figs. 7 and 8, show sizable values of A_N only at large z values, which might be difficult to reach.

Although the Sivers effect is not expected to contribute to A_N in lepton–nucleon interactions, unless some really new and important mechanism allows avoiding problems with time reversal invariance, we have computed A_N assuming only the Sivers effect to be at work, Figs. 9–12; a comparison with the corresponding results originated by the Collins effect, Figs. 1–4, shows interesting differences: in particular $|A_N|$ is biggest for π^+ according to the Collins mechanism, whereas it is biggest for π^- according to Sivers.

To conclude, we believe that the measurement of A_N – possibly in the simple and more direct channel $\ell p^\uparrow \to \pi X$

– can and should be performed by several experiments; the comparison with existing values of A_N from $p^{\uparrow}p \to \pi X$ and $\bar{p}^{\uparrow}p \to \pi X$ processes would immediately allow one to draw conclusions on the possible origins of single spin effects. Once more, spin dependent measurements probe the hadronic structure and the corresponding theories and models at deeper levels than the usual unpolarized quantities.

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